Answer Question No. 1 and any four from the rest

Time: 4 Hours   **Full Marks: 80**

1. Answer any eight questions: 8×2=16

   a) Evaluate: $\int \dfrac{x^2}{1+x^2}\,dx$.

   b) Evaluate: $\int \dfrac{dx}{x^2+2x+2}$.

   c) Evaluate: $\int \dfrac{x^4}{x^2+1}\,dx$.

   d) Define differential equation.

   e) Eliminate the arbitrary constants A and B from the relation: $y = Ae^x + Be^{-x} + x^2$.

   f) Determine the differential equations, whose primitive is y=ax+b.

   g) What do you mean by limit of a function?

   h) Find: $\displaystyle\lim_{x\to 0} \dfrac{x^2 \sin \tfrac{1}{x}}{\sin x}$

   i) Show that: $\displaystyle\lim_{x\to \pi} \dfrac{\sin x}{\pi - x} = 1$.

   j) Find: $\displaystyle\lim_{x\to \pi/4} (\sec 2x - \tan 2x)$

   k) Evaluate: $\displaystyle\lim_{x\to 1} \dfrac{x^2+2x-1}{2x+2}$.

   l) Define continuity in respect of Differential Calculus.

2. a) Solve: $x^2 \dfrac{dy}{dx} + y = 1$.   4

   b) Solve: $y - x\dfrac{dy}{dx} = a(y^2 + \dfrac{dy}{dx})$ 4

   c) Solve: $(x+y)^2 \dfrac{dy}{dx} = a^2$.   6

   d) Find the order and degree of $y = x\dfrac{dy}{dx} + \dfrac{dy}{dx} - (\dfrac{dy}{dx})^3$ 2

3. a) Solve: $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$.   4

b) Solve: $x\log x\,dy + \sqrt{1-y}$.   6

c) Find the particular solution of the equation:

$(1+y^2)dx - (y+\sqrt{1+y^2})(1+x)^{3/2}\,dy = 0$ if y=0 when x=1.   6

4. a) Evaluate: $\displaystyle\int \frac{dx}{x^2+x^3}$.   4

b) Integrate $\sqrt{(e^x-1)}$, with respect to x.   4

c) Evaluate: $\displaystyle\int \sqrt{(a^2-x^2)}\,dx$.   4

d) Evaluate: $\displaystyle\int \frac{x^4\,dx}{x^2-x+1}$.   4

5. a) Evaluate the integral $\int_0^2 (ax+b)dx$ using definite integral as a limit of a sum.

     8

b) Find domain and range of the function $f(x) = \dfrac{|x|}{x}$ 4

c) If $f(x) = \dfrac{|x|}{x}$ and c($\neq$0)be any real number, show that $|f(c)-f(-c)| = 2$ 4

6. a) Show that the sequence $\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}$ ......converges to 3.   8

b) Define Sandwich theorem for limit.   2

c) . If $x^y + y^x = 1$, then find $\dfrac{dy}{dx}$ 6

7. a) Prove that every convergent sequence is bounded.   4

b) If $x_n = \dfrac{1}{1.3} + \dfrac{1}{3.5} + \dfrac{1}{5.7} + ... + \dfrac{1}{(2n-1)(2n+1)}$, then show that the sequence $\{x_n\}$ is

monotonic increasing and bounded.   6

c) Find $\dfrac{dy}{dx}$, if $y = \sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)}}$ 6